LLM Sys

11868 LLM Systems

LLM Quantization - GPTQ

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Highlights from GTC 2025

NVIDIA Dynamo, A Low-Latency Distributed Inference Framework (we already cover inference acceleration and will cover more about serving later)

GB300 GPU

GPU Memory	**288 GB**

NVIDIA CUDA-X
about 300 GPU-accelerated microservices and libraries for AI, data processing, HPC

Quantize a Number to Int8

- Absmax quant

$$\mathbf{X}_{\text{quant}} = \text{round}\left(\frac{127}{\max|\mathbf{X}|} \cdot \mathbf{X}\right)$$

$$\mathbf{X}_{\text{dequant}} = \frac{\max|\mathbf{X}|}{127} \cdot \mathbf{X}_{\text{quant}}$$

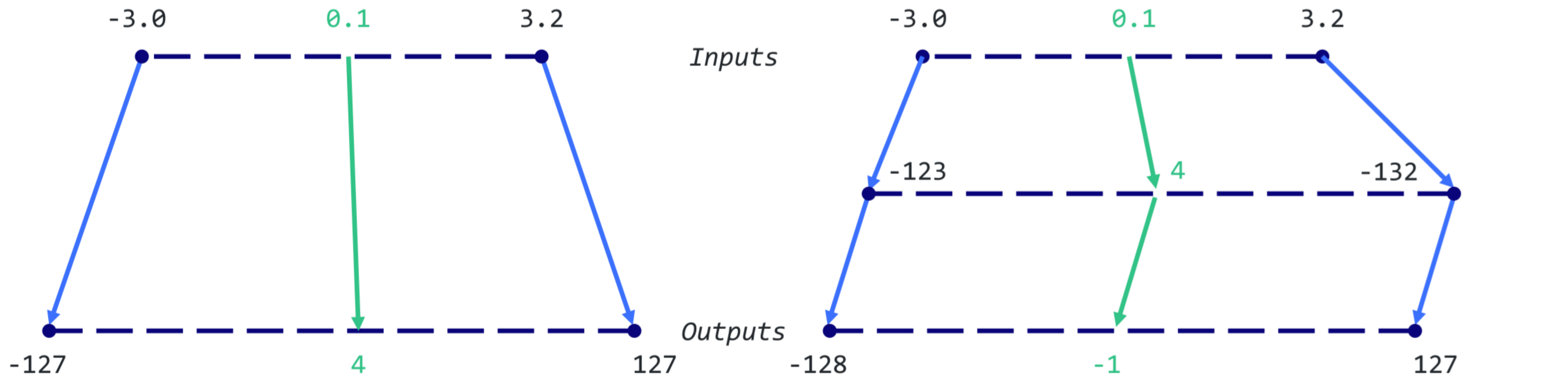
- Zero-point quant

$$\text{scale} = \frac{255}{\max(\mathbf{X}) - \min(\mathbf{X})}$$

$$\text{zeropoint} = -\text{round}(\text{scale} \cdot \min(\mathbf{X})) - 128$$

$$\mathbf{X}_{\text{quant}} = \text{round}\left(\text{scale} \cdot \mathbf{X} + \text{zeropoint}\right)$$

$$\mathbf{X}_{\text{dequant}} = \frac{\mathbf{X}_{\text{quant}} - \text{zeropoint}}{\text{scale}}$$



Outline

- GPTQ
- Code Walkthrough

Overall idea of GPTQ

- solving layer-wise quantization.

$$\underset{\widehat{W}}{\operatorname{argmin}} \left\| WX - \widehat{W}X \right\|_2^2$$

- Key idea:
  - Quantizes one column-block of weights at a time
  - Updates all the not-yet-quantized weights, to compensate for the error incurred by quantizing a single weight

GPTQ: Accurate Post-Training Quantization for Generative Pre-trained Transformers. Frantar et al. ICLR 2023.

Optimal Brain Surgeon and General Network Pruning (1993)

Optimal Brain Compression: A framework for accurate post-training quantization and pruning (2022)

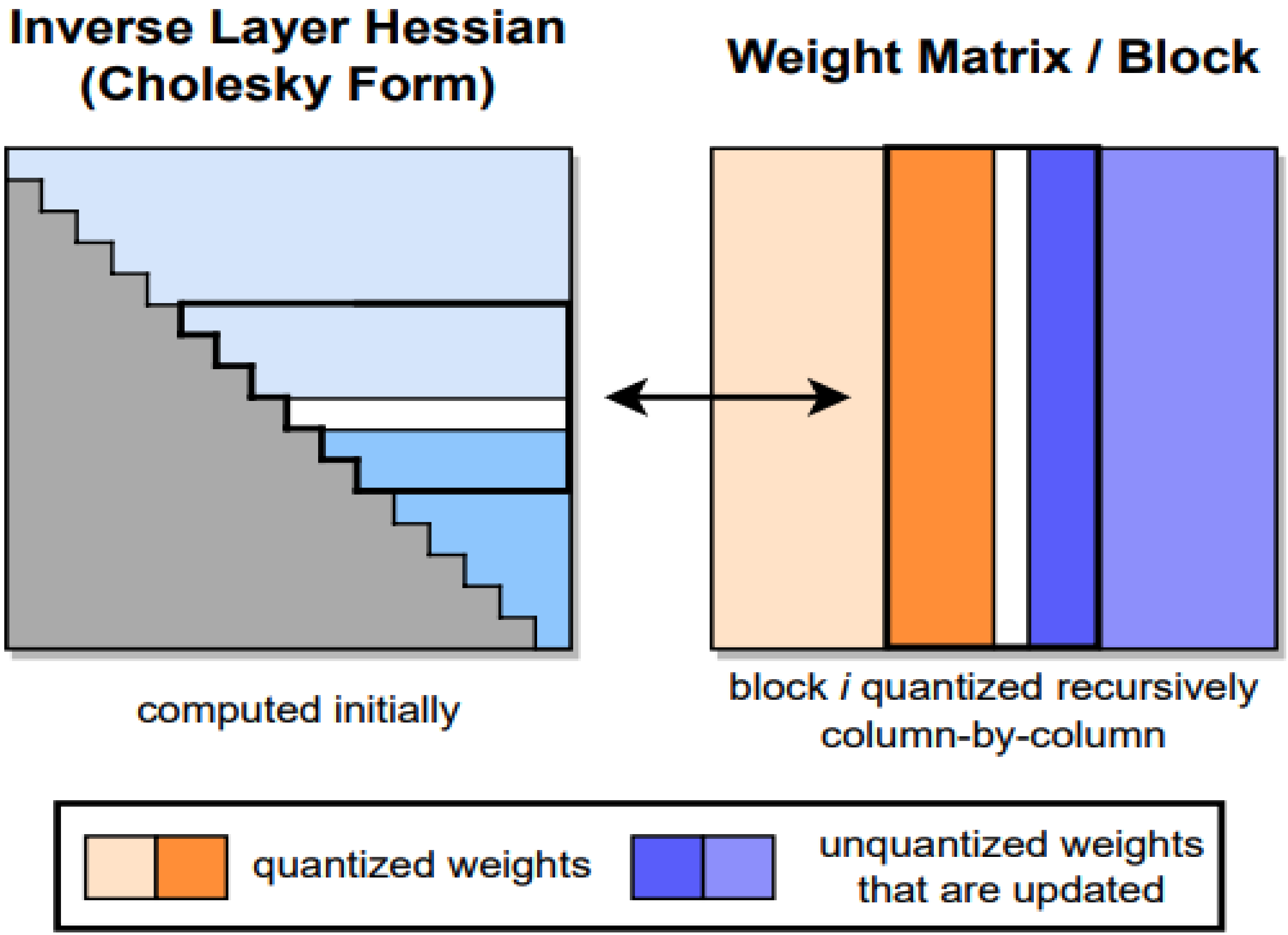
GPTQ algorithm

1. Pre-compute Cholesky decomposition of the Hessian inverse for input X

2. Iteratively handle one batch of columns of weights W
    1. it quantizes the weights using a specific rounding,
    2. calculates the rounding error
    3. updates the weights in the column block accordingly.
    4. After processing the batch, it updates all remaining weights based on the block's errors.

GPTQ: Pre-compute

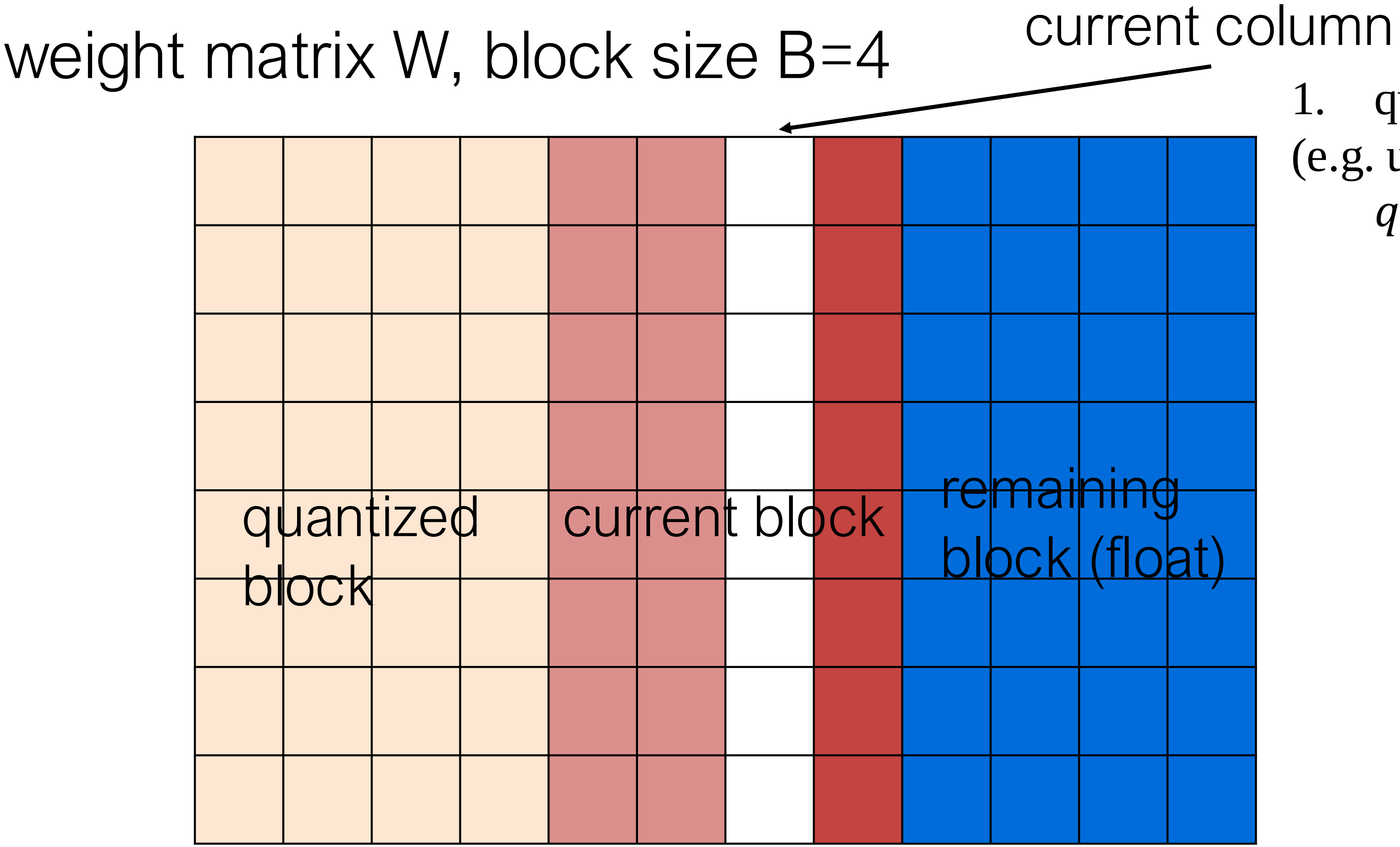
$$G = \text{Cholesky}((2X \cdot X^T + \lambda I)^{-1}))^T$$



Cholesky decomposition, given a symmetric positive-definite matrix A

$$A = L \cdot L^T$$

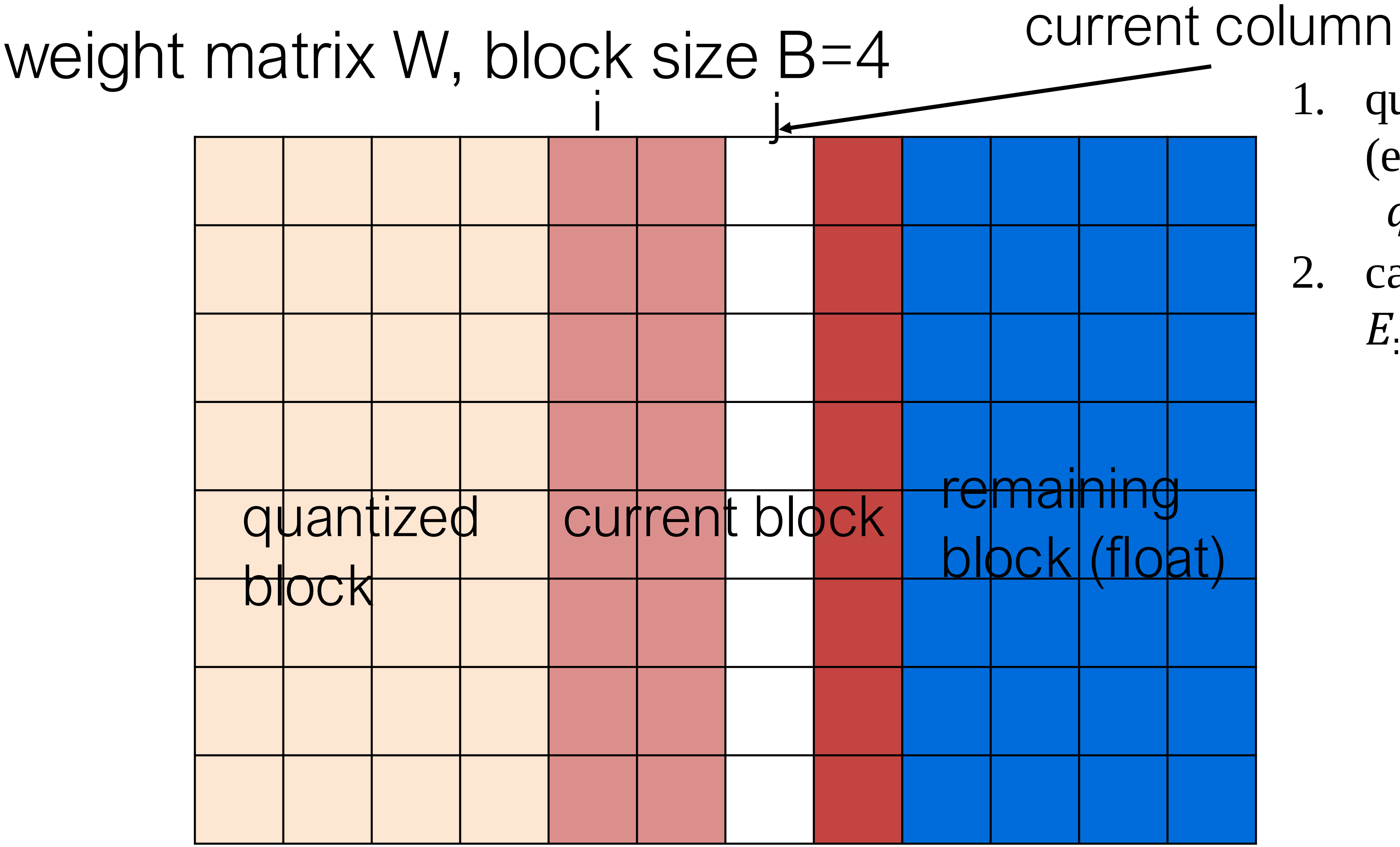
GPTQ: Block-wise Quantize and Update



1. quantize the column weights (e.g. using int8 or int4)

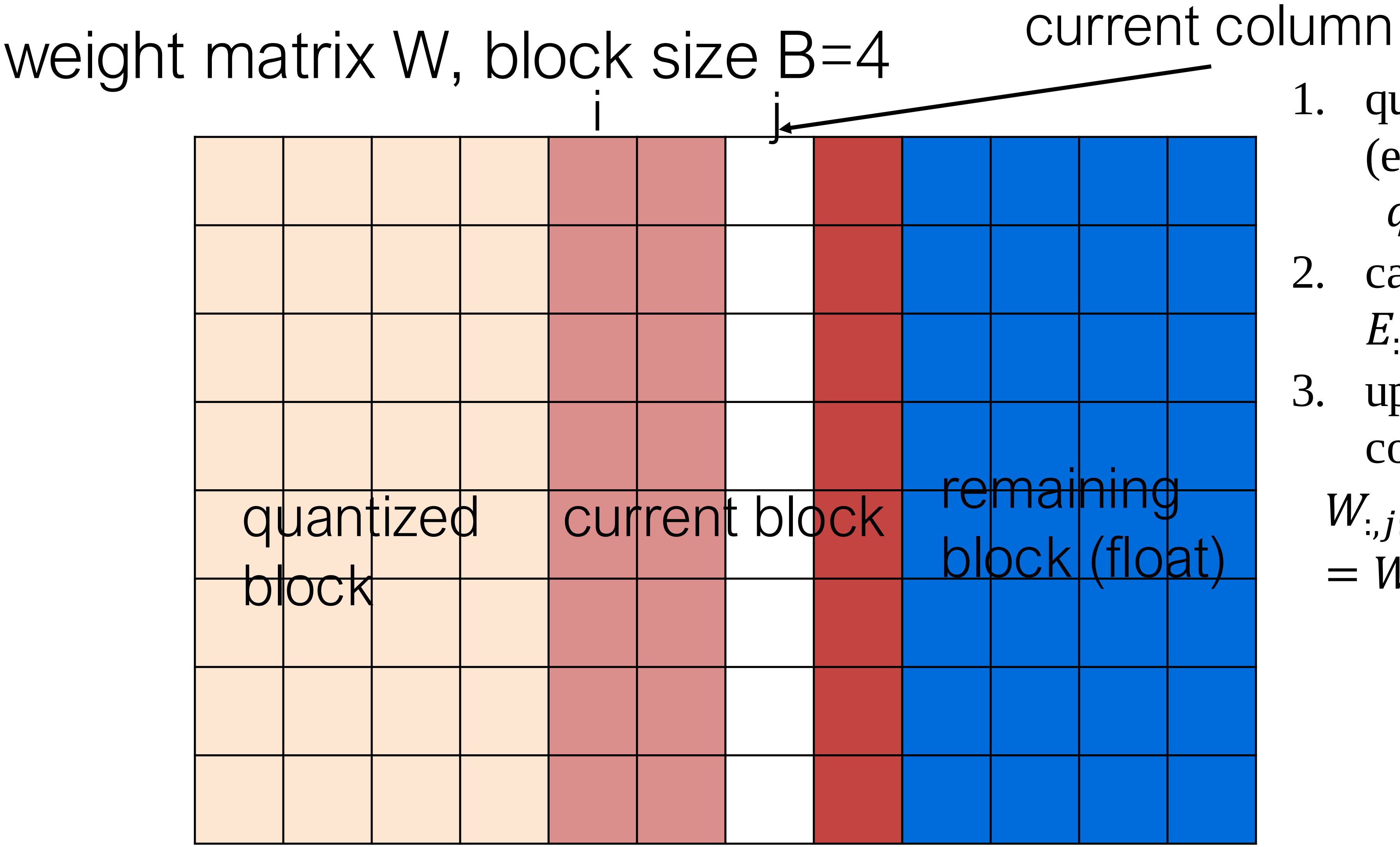
$$q_{:,j} = \text{quant}(W_{:,j})$$

GPTQ: Block-wise Quantize and Update



1. quantize the column weights (e.g. using int8 or int4)
   $q_{:,j} = \text{quant}(W_{:,j})$
2. calculates the rounding error
   $E_{:,j-i} = (W_{:,j} - Q_{:,j})/G_{j,j}$

GPTQ: Block-wise Quantize and Update



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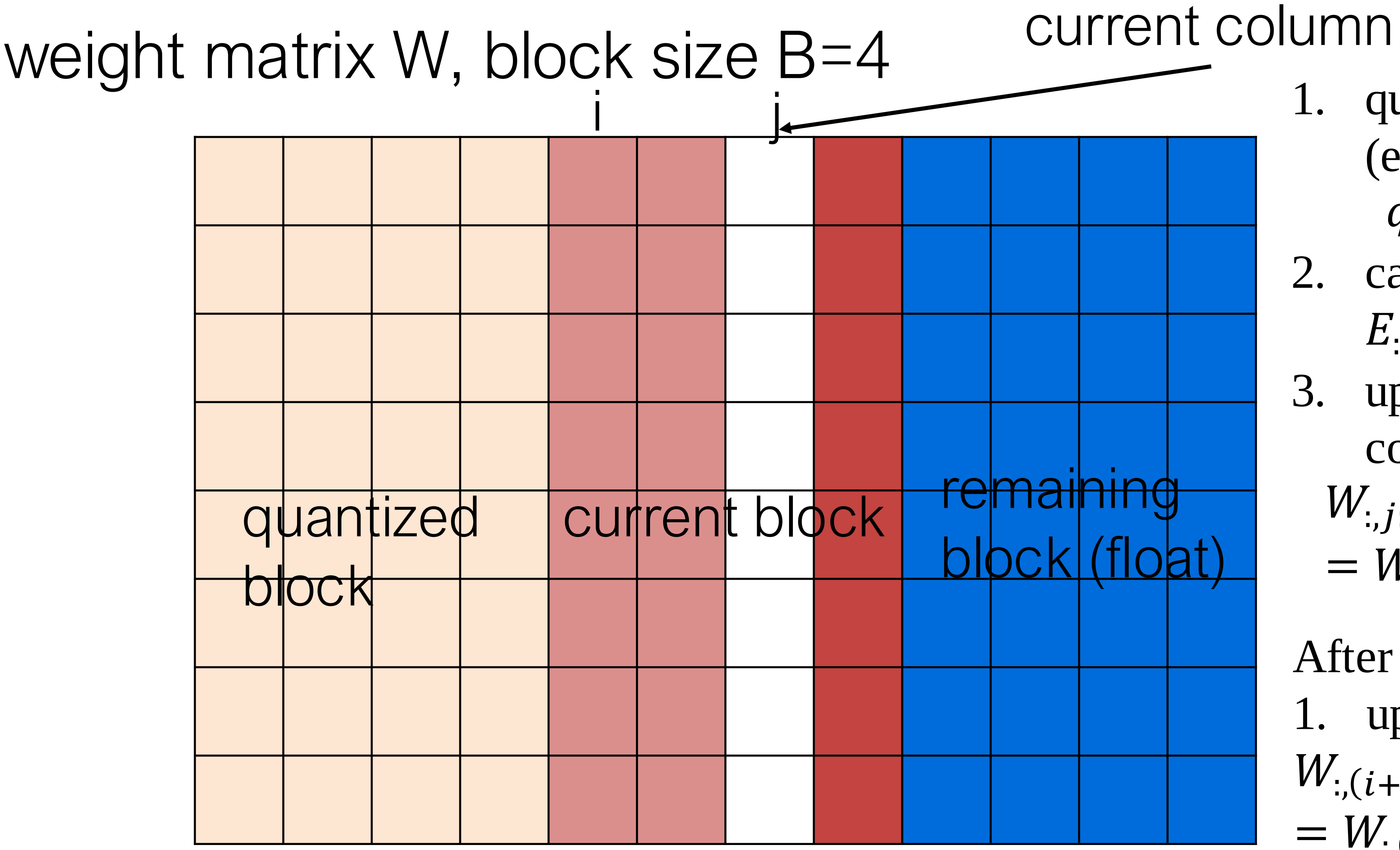
2. calculates the rounding error
   $E_{:,j-i} = (W_{:,j} - Q_{:,j})/G_{j,j}$
3. updates the weights in the column block

$$W_{:,j:(i+B)} = W_{:,j:(i+B)} - E_{:,j-i} \cdot G_{j,j:(i+B)}$$

GPTQ: Lazy-update for rest weights



1. quantize the column weights (e.g. using int8 or int4)
   $q_{:,j} = \text{quant}(W_{:,j})$
2. calculates the rounding error
   $E_{:,j-i} = (W_{:,j} - Q_{:,j})/G_{j,j}$
3. updates the weights in the column block
   $W_{:,j:(i+B)} = W_{:,j:(i+B)} - E_{:,j-i} \cdot G_{j,j:(i+B)}$

After compute the current block

1. update remaining weights

$W_{:,(i+B):} = W_{:,(i+B):} - E \cdot G_{i:(i+B),(i+B):}$

GPTQ Algorithm

Algorithm 1 Quantize $\mathbf{W}$ given inverse Hessian $\mathbf{H}^{-1} = (2\mathbf{X}\mathbf{X}^\top + \lambda\mathbf{I})^{-1}$ and blocksize B.

$\mathbf{Q} \leftarrow \mathbf{0}_{d_{\text{row}} \times d_{\text{col}}}$ *// quantized output*
$\mathbf{E} \leftarrow \mathbf{0}_{d_{\text{row}} \times B}$ *// block quantization errors*
$\mathbf{H}^{-1} \leftarrow \text{Cholesky}(\mathbf{H}^{-1})^\top$ *// Hessian inverse information* =G in previous slides
for $i = 0, B, 2B, \ldots$ **do**
  for $j = i, \ldots, i + B - 1$ **do**
    $\mathbf{Q}_{:,j} \leftarrow \text{quant}(\mathbf{W}_{:,j})$ *// quantize column*
    $\mathbf{E}_{:,j-i} \leftarrow (\mathbf{W}_{:,j} - \mathbf{Q}_{:,j}) \, / \, [\mathbf{H}^{-1}]_{jj}$ *// quantization error*
    $\mathbf{W}_{:,j:(i+B)} \leftarrow \mathbf{W}_{:,j:(i+B)} - \mathbf{E}_{:,j-i} \cdot \mathbf{H}^{-1}_{j,j:(i+B)}$ *// update weights in block*
  end for
  $\mathbf{W}_{:,(i+B):} \leftarrow \mathbf{W}_{:,(i+B):} - \mathbf{E} \cdot \mathbf{H}^{-1}_{i:(i+B),(i+B):}$ *// update all remaining weights*
end for

Why GPTQ works?

- Quantize one weight in W can be solved using the Optimal Brain Surgeon method (OBS)
  - ➔ We can update one column of weights
- Iteratively updating the inverse Hessian can be updated efficiently (rank-1 update using Optimal Brain Quantization, OBQ method)
  - ➔ Using Cholesky to pre-compute
- Updating weights after calculating rounding errors can be done in batch and lazy-fashion

Optimal Brain Surgeon

- Taylor approximation to find optimal single weight to remove, and optimal update of remaining weights to compensate.

- Weight to prune ω_p which incurs the minimal increase in loss and the corresponding update of the remaining weights δ_p is,

$$\omega_p = \underset{\omega_p}{argmin}\, \omega_p \frac{\omega_p^2}{[H^{-1}]_{pp}}, \delta_p = -\frac{\omega_p}{[H^{-1}]_{pp}} \cdot H_{:,p}^{-1},$$

  In transformers, ω_p could potentially be LayerNorm/FeedForward weights

- H is a d×d Hessian matrix where $d = d_{row} \cdot d_{col}$, which is expensive to store and compute with.

- H needs to be updated and inverted at O(d) pruning steps with a Θ(d^3) complexity. Total runtime O(d^4) is too inefficient.

Optimal Brain Quantization

- OBQ picks the greedy optimal weight w_q to quantize next, along with the update δ_F to all unquantized weights in F.

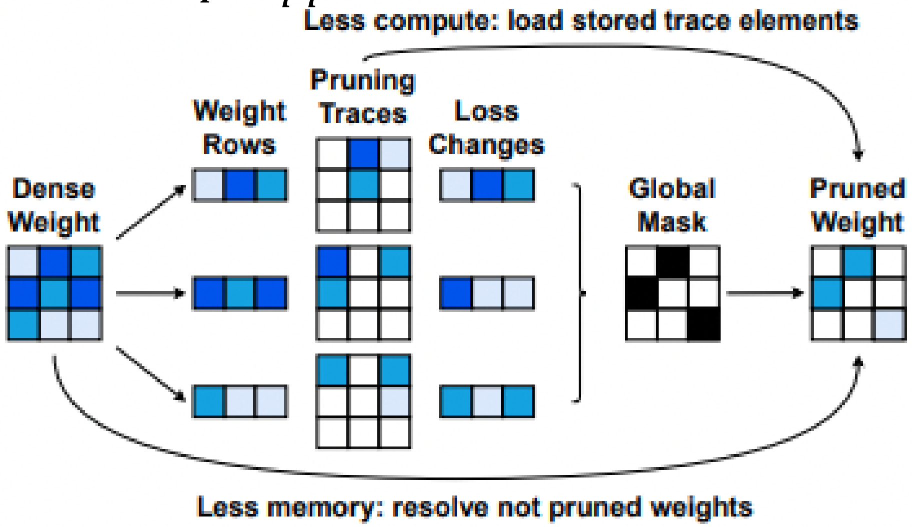
$$w_q = \text{argmin}\, w_q \frac{(quant(w_q) - w_q)^2}{[H_F^{-1}]_{pp}}, \delta_F = -\frac{w_q - quant(w_q)}{[H_F^{-1}]_{pp}} \cdot (H_F^{-1})_{:,q}.$$

full-precision

update

- quantizes weights iteratively until all weights are quantized.

- Hessian $H = 2XX^T$



Optimal Brain Quantization

1. Row wise decomposition: OBQ applies OBS per row of the weight matrix

$$\sum_{i=1}^{d_{row}} \left\| W_{i,:}X - \widehat{W_{i,:}}X \right\|_2^2$$

   No Hessian interaction between different rows and so we can work with the individual $d_{col} \times d_{col}$ H corresponding to each of the d_{row} rows.

2. Efficient inverse

   Reduces overall costs of this process to $O(d_{row} \cdot d_{col}^3)$ time and $O(d_{col}^2)$ memory.

$$\mathbf{H}_{-q}^{-1} = \left(\mathbf{H}^{-1} - \frac{1}{[\mathbf{H}^{-1}]_{qq}} \mathbf{H}_{:,q}^{-1} \mathbf{H}_{q,:}^{-1} \right)_{-p}$$

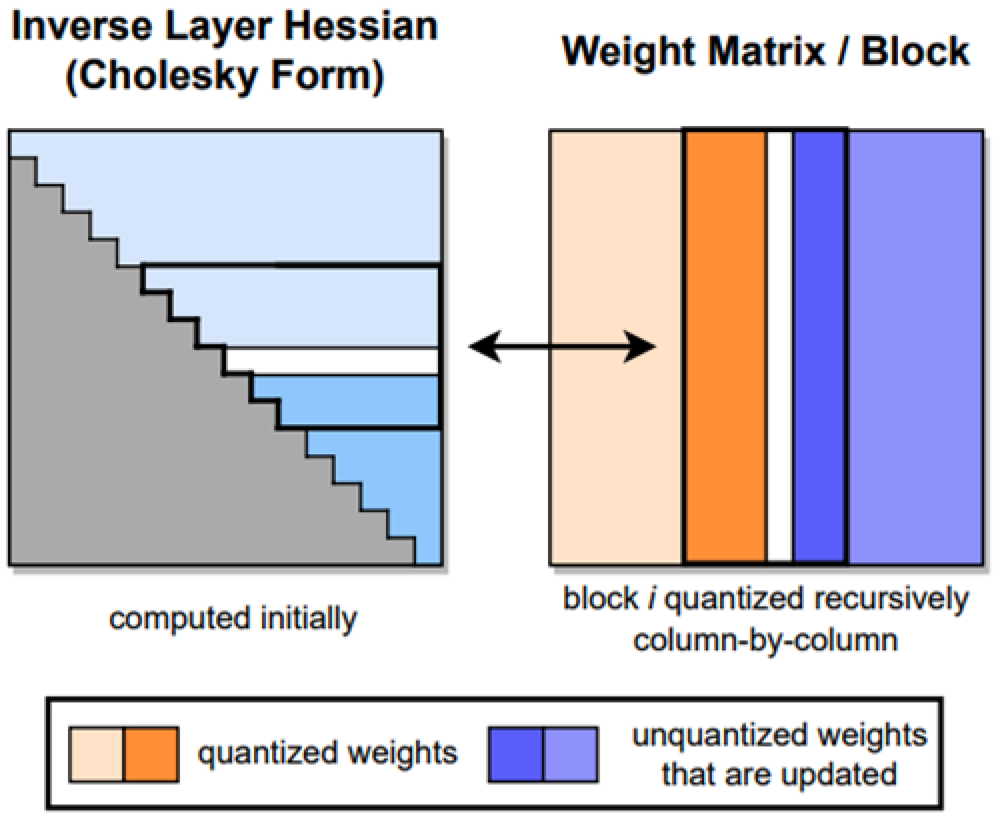
Column Update using OBQ

- Quantize all rows of **weights in same order**.
  - ➔ column-wise update

- F and H_F^{-1} are always the same for all rows as H_F depends only on the layer inputs X_F, which are the same for all rows.

- Perform update on H_F^{-1} only d_{col} times, once per column, rather than $d_{row} \cdot d_{col}$ times, once per weight.

- This reduces the overall runtime from $O(d_{row} \cdot d_{col}^3)$ to $O(max(d_{row} \cdot d_{col}^2, d_{col}^3))$.



Insight of Arbitrary Update Order for OBQ

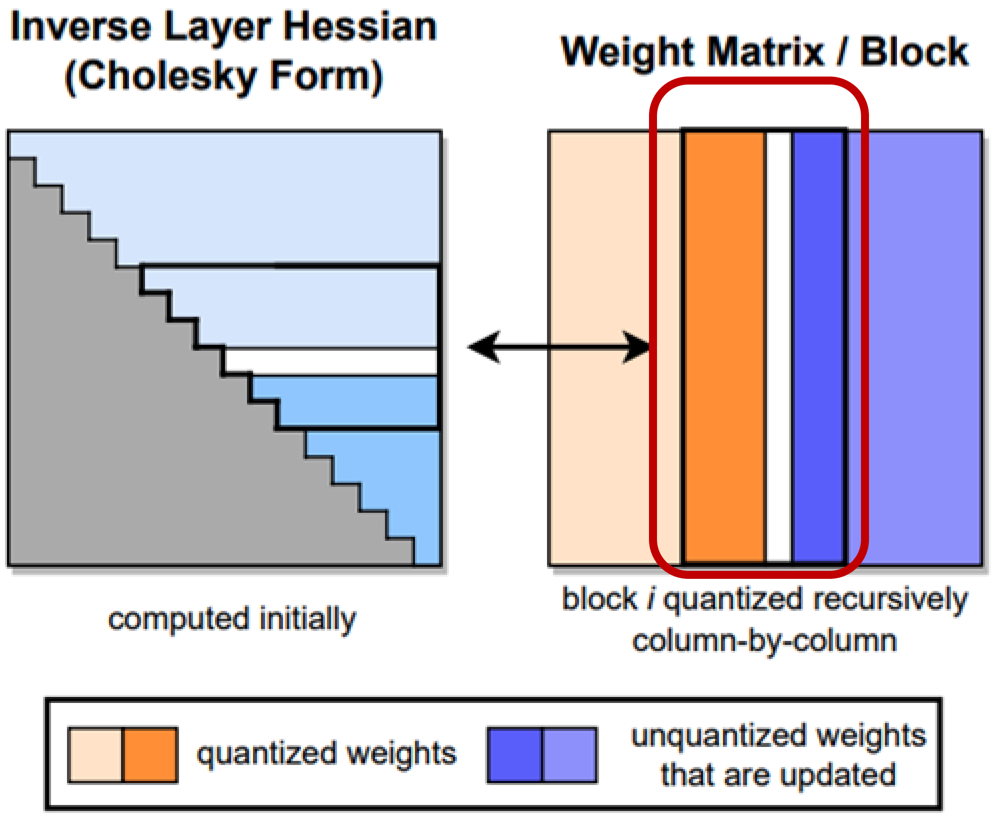
- OBQ quantizes weights in a specific order defined by the corresponding errors.
- Improvement over quantizing the weights in arbitrary order is generally small
  - quantized weights with large individual error is balanced out by those weights towards the end of the process
  - few other unquantized weights can be adjusted for compensation

GPTQ: Accurate Post-Training Quantization for Generative Pre-trained Transformers. Frantar et al. ICLR 2023.

Lazy Batch Updates

- Naïve column update is not fast in practice
  - **low compute-to-memory-access ratio**
  - cannot highly utilize GPUs compute.

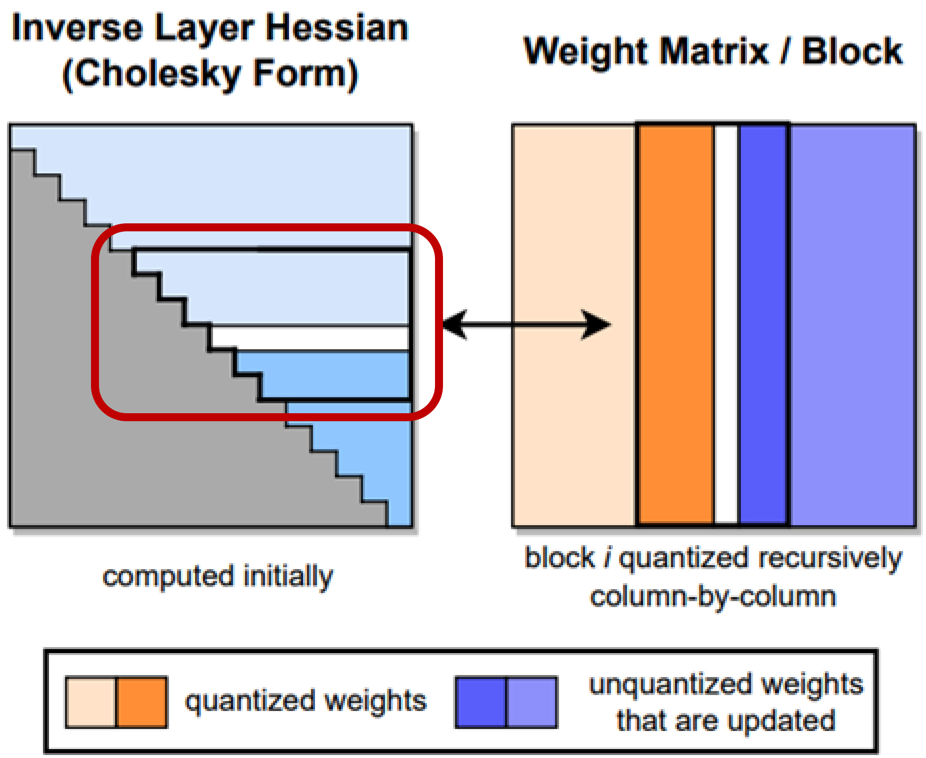
- Observation:
  - Rounding decisions for col i only affected by updates on this col
  - Updates to later columns are irrelevant at this point in the process.
- Efficient update



$$W_{:,(i+B):} = W_{:,(i+B):} - E \cdot G_{i:(i+B),(i+B):}$$

Cholesky Pre-computation

- **Numerical inaccuracies, can become a major problem** at the scale of LLMs,
- H_F^{-1} can become indefinite
- Observation:
  - Only information required from $H_{F_q}^{-1}$ when quantizing weight q from unquantized F_q, are the elements in row q starting with the diagonal.
- GPTQ leverages **Cholesky kernels** to precompute all information from H^{-1} without any significant increase in memory consumption.



Research Questions

- How is GPT-Q's perf on small models compared with accurate-but-expensive methods?
- How does GPT-Q's quantization time scale with model size?
- How is GPT-Q's perf on large models compared with Round-to-nearest methods?
- How does GPT-Q speed up model inference in practical applications?
- Does GPT-Q even work for extreme 2-bit quantization?

Experiment Setup

- Calibration data randomly sampled from C-4 dataset to ensure GPT-Q is not task-aware.
- Standard uniform per-row asymmetric quantization on the min-max grid
- Quantize on each transformer block (6 layers), with input X from last quantized block output.

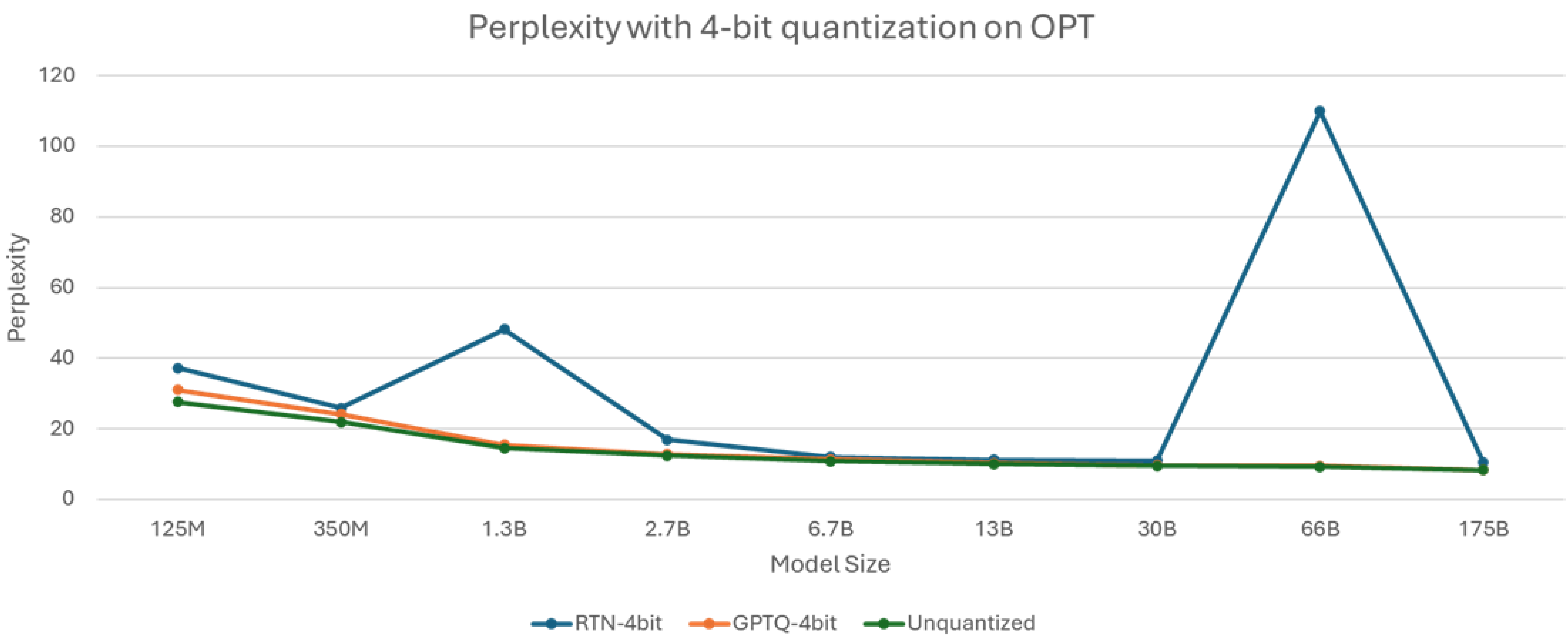
How does GPT-Q speed up model inference in practical applications?

OPT-175B mode (Measured with pipeline parallelism)

GPU	FP16	3bit	Speedup	GPU reduction
A6000 – 48GB	589ms	130ms	4.53×	$8 \rightarrow 2$
A100 – 80GB	230ms	71ms	3.24×	$5 \rightarrow 1$

- Single-batch inference is memory-bound because of GEMVs. Although dequantization consumes extra compute, the custom kernel reduces memory access and thus reduces e2e time.

How is GPT-Q's perf on large models compared with Round-to-nearest methods?



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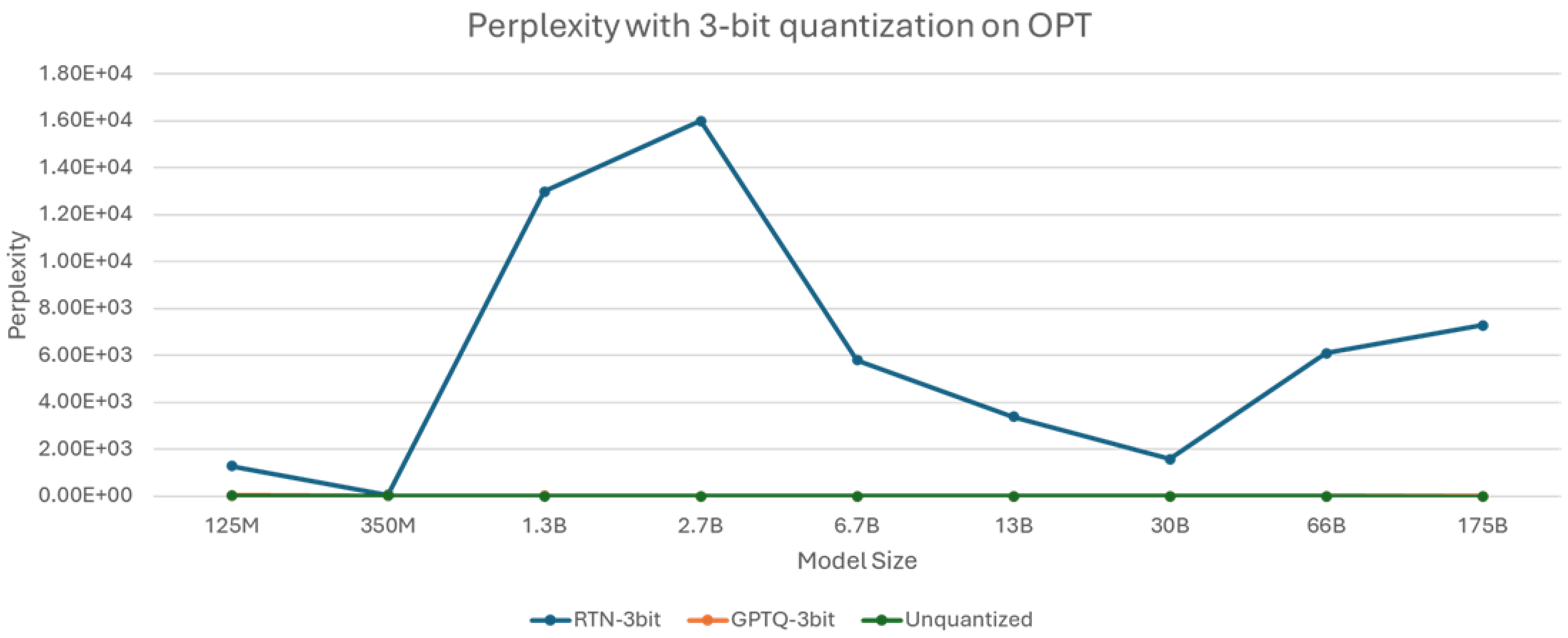
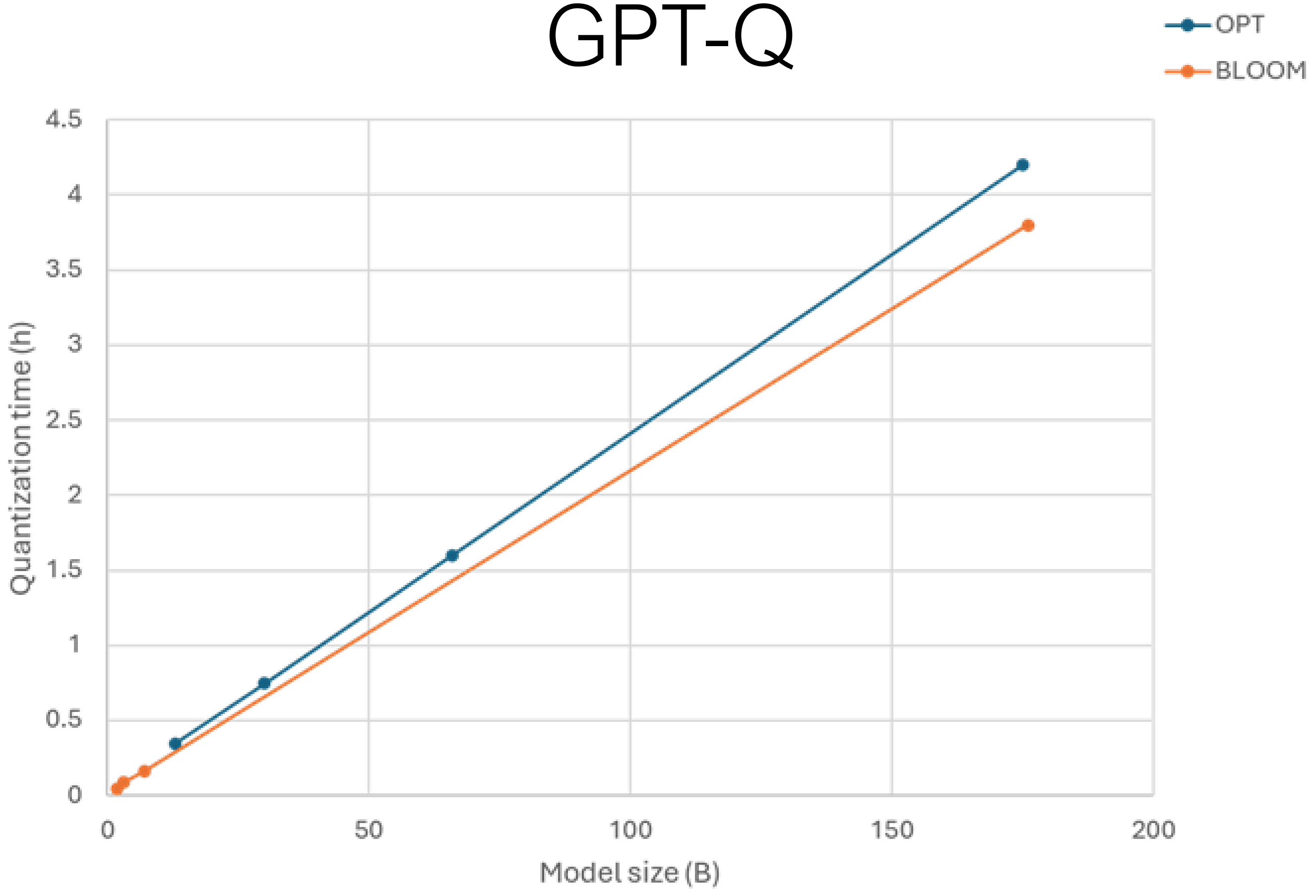


How does GPT-Q's quantization time scale with model size?

GPT-Q

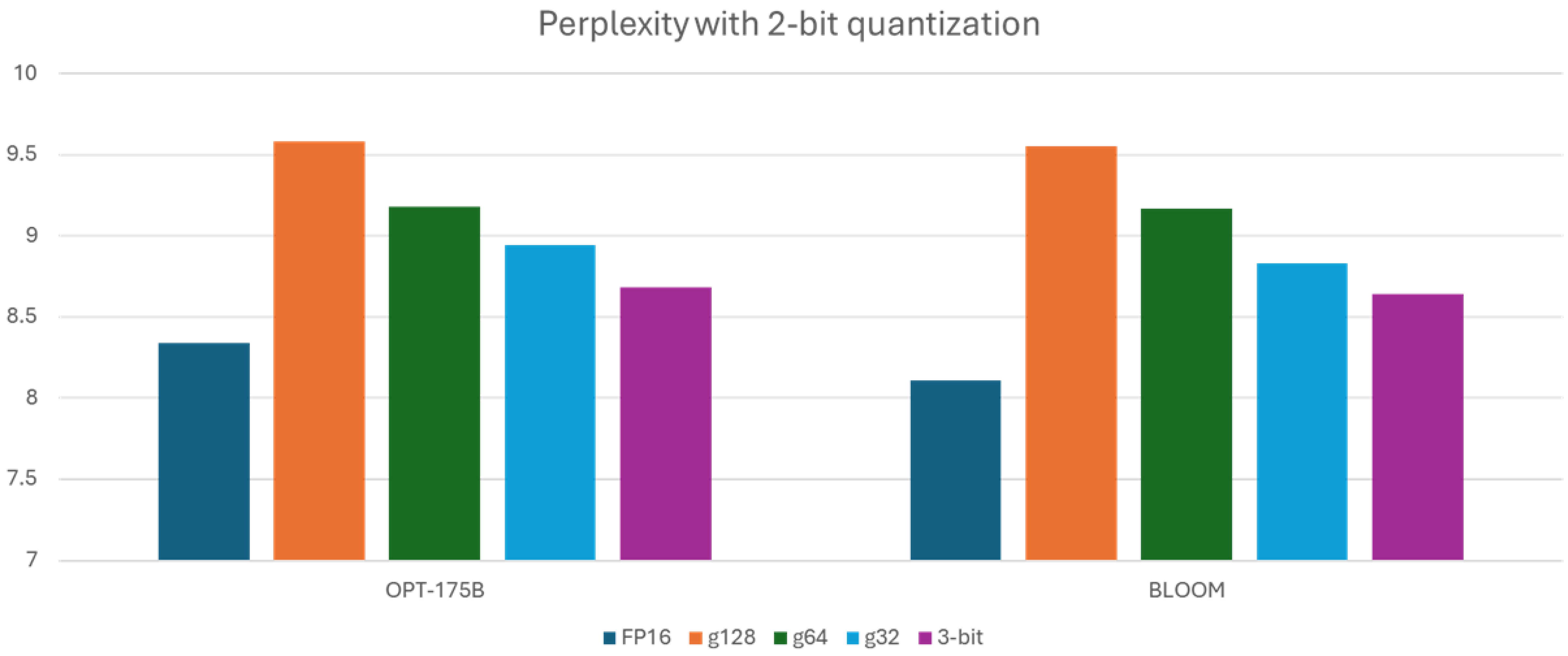


ZeroQuant-LKD

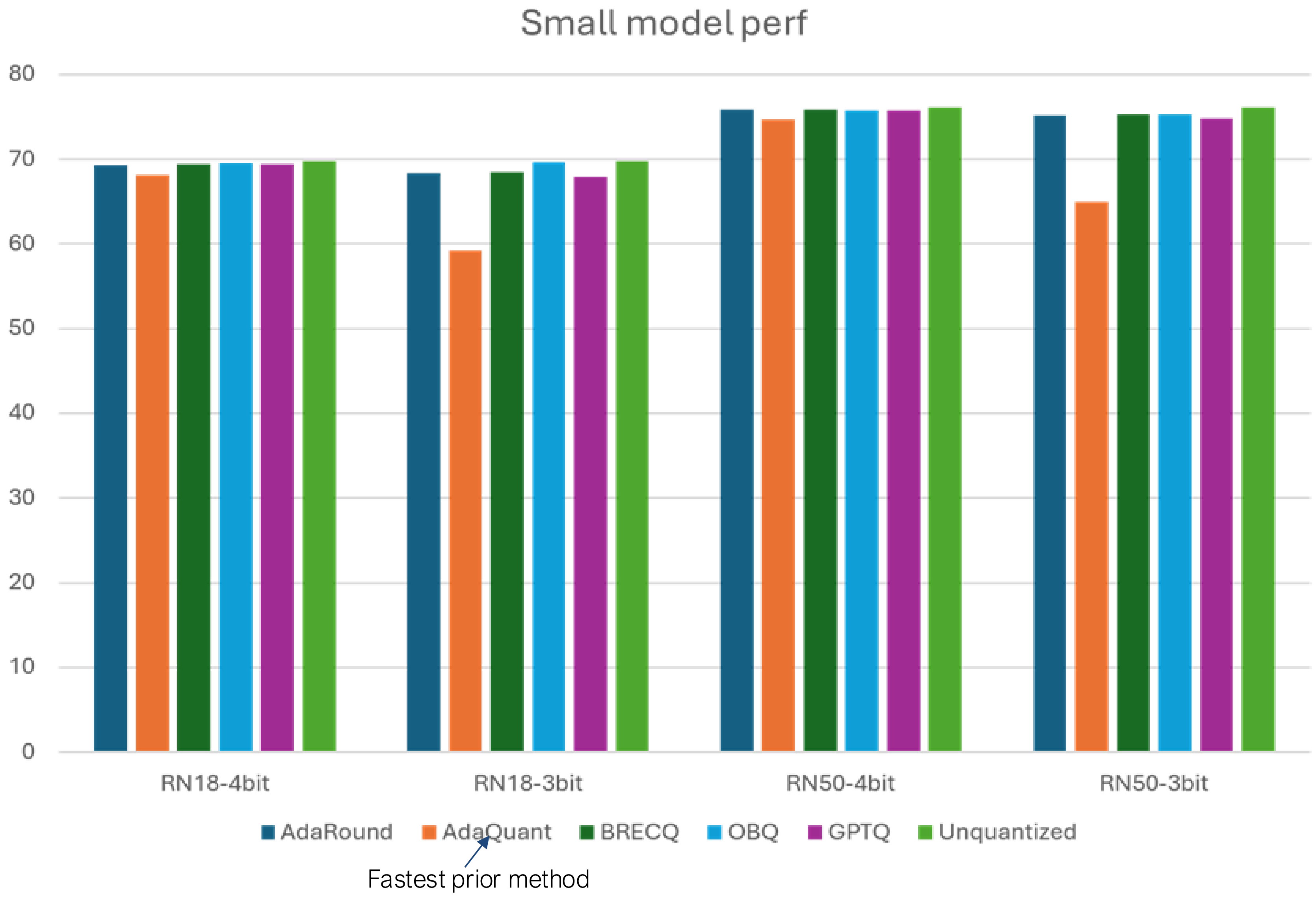
1.3B model - 3h

* Measured on single A100

Does GPT-Q even work for extreme 2-bit quantization?



How is GPT-Q's perf on small models compared with accurate-but-expensive methods?



Quiz 9

- https://canvas.cmu.edu/courses/44373/quizzes/141974

GPTQ for LLaMA

- https://github.com/qwopqwop200/GPTQ-for-LLaMa/
- GPTQ in
    - https://github.com/qwopqwop200/GPTQ-for-LLaMa/blob/triton/gptq.py

GPTQ: Initialization

```
def __init__(self, layer):
    self.layer = layer
    self.dev = self.layer.weight.device
    W = layer.weight.data.clone()
    if isinstance(self.layer, nn.Conv2d):
        W = W.flatten(1)
    if isinstance(self.layer, transformers.Conv1D):
        W = W.t()
    self.rows = W.shape[0]
    self.columns = W.shape[1]
    self.H = torch.zeros((self.columns, self.columns), device=self.dev)
    self.nsamples = 0
```

- Reshape weights from the input layer
- Initialize Hessian matrix

GPTQ: Hessian Matrix Update

```
def add_batch(self, inp, out):
  if len(inp.shape) == 2:
      inp = inp.unsqueeze(0)
  tmp = inp.shape[0]
  if isinstance(self.layer, nn.Linear) or isinstance(self.layer, transformers.Conv1D):
      if len(inp.shape) == 3:
          inp = inp.reshape((-1, inp.shape[-1]))
      inp = inp.t()
  if isinstance(self.layer, nn.Conv2d):
      unfold = nn.Unfold(
          self.layer.kernel_size,
          dilation=self.layer.dilation,
          padding=self.layer.padding,
          stride=self.layer.stride
      )
      inp = unfold(inp)
      inp = inp.permute([1, 0, 2])
      inp = inp.flatten(1)
  self.H *= self.nsamples / (self.nsamples + tmp)
  self.nsamples += tmp
  # inp = inp.float()
  inp = math.sqrt(2 / self.nsamples) * inp.float()
  # self.H += 2 / self.nsamples * inp.matmul(inp.t())
  self.H += inp.matmul(inp.t())
```

- Update Hessian matrix with information from a new batch of the input and output pairs

GPTQ: Lazy Batch-Update

```
for i1 in range(0, self.columns, blocksize):
    i2 = min(i1 + blocksize, self.columns)
    count = i2 - i1

    W1 = W[:, i1:i2].clone()
    Q1 = torch.zeros_like(W1)
    Err1 = torch.zeros_like(W1)
    Losses1 = torch.zeros_like(W1)
    Hinv1 = Hinv[i1:i2, i1:i2]

    for i in range(count):
        w = W1[:, i]
        d = Hinv1[i, i]

        if groupsize != -1:
            if not static_groups:
                if (i1 + i) % groupsize == 0:
                    self.quantizer.find_params(W[:, (i1 + i):(i1 + i + groupsize)], weight=True)
            else:
                idx = i1 + i
                if actorder:
                    idx = perm[idx]
                self.quantizer = groups[idx // groupsize]
```

- Processes weight matrix W in blocks.
- Updates quantization parameters conditionally based on group size and static grouping settings.

GPTQ: Lazy Batch-Update

```
    q = quantize(
        w.unsqueeze(1), self.quantizer.scale, self.quantizer.zero, self.quantizer.maxq
    ).flatten()
    Q1[:, i] = q
    Losses1[:, i] = (w - q) ** 2 / d ** 2

    err1 = (w - q) / d
    W1[:, i:] -= err1.unsqueeze(1).matmul(Hinv1[i, i:].unsqueeze(0))
    Err1[:, i] = err1

Q[:, i1:i2] = Q1
Losses[:, i1:i2] = Losses1 / 2

W[:, i2:] -= Err1.matmul(Hinv[i1:i2, i2:])
```

- Applies quantization function quantize to weights and computes the loss due to quantization.
- Adjusts remaining block weights based on quantization error to minimize the overall error.

GPTQ: Cholesky Reformulation

```
damp = percdamp * torch.mean(torch.diag(H))
diag = torch.arange(self.columns, device=self.dev)
H[diag, diag] += damp
H = torch.linalg.cholesky(H)
H = torch.cholesky_inverse(H)
H = torch.linalg.cholesky(H, upper=True)
Hinv = H
```

- Applies damping to the Hessian matrix diagonals
- Performs Cholesky decomposition and inversion
- Transforms the Hessian into its inverse.

AutoGPTQ Tool

```
import random
from auto_gptq import AutoGPTQForCausalLM, BaseQuantizeConfig
from datasets import load_dataset
import torch
from transformers import AutoTokenizer

# Define base model and output directory
model_id = "gpt2" #modify to your model
out_dir = model_id + "-GPTQ"

# Load quantize config, model and tokenizer
quantize_config = BaseQuantizeConfig(bits=4, group_size=128, damp_percent=0.01, desc_act=False)
model = AutoGPTQForCausalLM.from_pretrained(model_id, quantize_config)
tokenizer = AutoTokenizer.from_pretrained(model_id)

# Load data and tokenize examples
n_samples = 1024
data = load_dataset("allenai/c4", data_files="en/c4-train.00001-of-01024.json.gz", split=f"train[:{n_samples*5}]")
tokenized_data = tokenizer("\n\n".join(data['text']), return_tensors='pt')

# Format tokenized examples
examples_ids = []
for _ in range(n_samples):
i = random.randint(0, tokenized_data.input_ids.shape[1] - tokenizer.model_max_length - 1)
j = i + tokenizer.model_max_length
input_ids = tokenized_data.input_ids[:, i:j]
attention_mask = torch.ones_like(input_ids)
examples_ids.append({'input_ids': input_ids, 'attention_mask': attention_mask})

# Quantize with GPTQ
model.quantize(examples_ids, batch_size=1, use_triton=True)

# Save model and tokenizer
model.save_quantized(out_dir, use_safetensors=True)
tokenizer.save_pretrained(out_dir)
```

Summary and Limitations

- GPTQ
  - approximate second-order of weights
  - accurately compress some of the largest publicly-available models down to 3 and 4 bits, and bring end-to-end speedups
- Limitations
  - Theoretical computation is the same
  - Focus on weight quantization, and does not consider activation quantization